On stochastic switching of bistable resonant-tunneling structures via nucleation

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We estimate the critical size of the initial nucleus of the low current state in a bistable resonant tunneling structure which is needed for this nucleus to develop into a lateral switching front. Using the results obtained for deterministic switching fronts, we argue that for realistic structural parameters the critical nucleus has macroscopic dimensions and therefore is too large to be created by stochastic electron noise.

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In Refs. 1,2, the following switching mechanism has been discussed for the double-barrier resonant tunneling structure (DBRT) in presence of electron shot noise. A bistable DBRT with Z–shaped current–voltage characteristic is considered. The bistability is due to the charge accumulation in the quantum well.[3] The high current and low current states correspond to the high and low electron concentration n in the quantum well, respectively. The high current state, which is stable in absence of fluctuations, becomes metastable due to electron shot noise when the voltage V is chosen close to the threshold voltage $V_{\rm th}$ at the upper boundary of the bistability range (see Fig. 1 in Ref. 1,2). Hence the system eventually jumps to the low current state. Whereas in small structures this occurs uniformly over the whole area of the device,[4] in large area structures the transition may occur via *nucleation*. The nucleation is a two-stage process: First the transition happens in a small part of the device, forming an initial nucleus of the new state. Then this initial nucleus expands, leading to the transition of the whole structure to the new state. (This mechanism has been originally discussed with respect to bistable microstructures in Ref. 4.) However, in analogy with the well-known case of an equilibrium phase transition, to enable expansion of the initial nucleus of the new state, its lateral size r should exceed a certain critical size $r_{\rm cr}$.[4,5,6] Consequently, a quantitative estimate of $r_{\rm cr}$ would be useful in order to understand the relevance of the nucleation switching scenario in a DBRT.

The expansion of the nucleus represents a deterministic process of switching front propagation. Such nonlinear fronts in bistable DBRT have been studied in Refs. 7,8,9,10,11,12,13. Refs. 9,10,11,12,13 are specifically devoted to the sequential tunneling regime, considered in Ref. 1. The critical size $r_{\rm cr}$ can be estimated on the basis of these results. We shall focus on the lower bound for $r_{\rm cr}$ which is given by the characteristic diffusion length $\ell_{\rm D}$ of the spatially distributed bistable system.[4,5,6] Nuclei of smaller size $r < \ell_{\rm D}$ disappear due the lateral spreading of the electron charge in the quantum well and do not trigger a switching front. The effect of curvature should also be taken into account in case of two lateral dimensions as considered in Ref. 1,2, when the initial nucleus is cylindrical. This generally makes $r_{\rm cr}$ larger than $\ell_{\rm D}$.[5] However, regardless of the lateral dimensionality $\ell_{\rm D}$ corresponds to the lower bound for $r_{\rm cr}$.

Let us start with a simple analytical estimate for the front width. For a bistable DBRT, $\ell_{\rm D}$ is determined by the lateral spreading of the electron concentration in the well and the balance of the emitter-well and well-collector currents in the vertical (cathode-anode) direction which determines the regeneration of the stored electron charge. In the sequential tunneling regime the lateral spreading of electrons in the quantum well is dominated by an electron drift in the self-induced lateral electrical field.[11,13] It is known from the theory of pattern formation in active systems[5,6] that $\ell_{\rm D}$ is close to the width of the interface which connects the coexisting on and off state in the stationary or moving current density pattern such as a current density filament or front. The order of magnitude of the velocity of the switching front is given by (Eq. (19) in Ref. 13)

$$v \sim \sqrt{\frac{\mu \Gamma_L E_e^F}{e\hbar}}, \tag{1}$$

where μ is the electron mobility in the well, $\Gamma_L/\hbar$ is the tunneling rate via the emitter-well barrier, E_e^F is the Fermi level in the emitter, and e is the electron charge. The concentration of stored electrons in the front can be roughly approximated by a piecewise exponential profile (Eq. (A5),(A6) in Ref. 13). The characteristic "decay length" ℓ_W of this profile is of the order of

$$\ell_W \sim \frac{\hbar}{\Gamma_L}\, v = \sqrt{\frac{\mu \hbar E_e^F}{e \Gamma_L}}, \tag{2}$$

as immediately follows from Eqs. (17),(19),(A5),(A6) in Ref. 13. The front width W, defined as the width over which the electron concentration changes by approximately 95% of the high-to-low ratio, is related to ℓ_W as $W \approx 3\ell_W$. For typical values $\mu \sim 10^5\,{\rm cm}^2/{\rm V\,s}$,

$\Gamma_L \sim 1\,\mathrm{meV}$, $E_e^F \sim 10\,\mathrm{meV}$ we obtain $v \sim 10^7$ cm/s and $W \sim 1\ \mu$m.

Eq. (2) gives the characteristic scale of the front width and reveals its dependence on the main structural parameters. A quantitive evaluation of W follows from numerical simulations[13] which show that W can be larger than predicted by Eq. (2). According to Ref. 13 the front width is about 10 μm for a stationary front (the structural parameters: $\mu \sim 10^5\,\mathrm{cm^2/V\,s}$; $\Gamma_L \sim 0.5\,\mathrm{meV}$, $\Gamma_R \sim 0.1\,\mathrm{meV}$), it increases with voltage, i.e. for a moving front, and becomes several times larger near the threshold voltage V_{th}, i.e. at the end of the range of bistability (see Fig. 4b in Ref. 13, where a stationary front ($v = 0$) according to Fig. 4a corresponds to a voltage $|u| \approx 370\,\mathrm{mV}$, and $V_{\mathrm{th}} = |u_{\mathrm{th}}| \approx 410\mathrm{mV}$ according to Fig. 2 therein). Note that the mobility μ in the well depends on the scattering time and therefore is related to the broadening of the quasibound state in the quantum well. Since the bistability range of the DBRT structure shrinks and eventually disappears when the broadening of the quasibound state increases, it is not possible to substantially decrease ℓ_{D} by choosing a low mobility μ.

Another limitation imposed on r_{cr} is related to the vertical thickness of the tunneling structure w. The dependence of the energy of the quasibound state on the stored electron charge $\Delta E = e^2 n/C$ (Eq. 11 in Ref. 1, where C is the effective capacitance of the well) is applicable only for variations of n whose characteristic length λ is much larger than w.[7,11] Variations of n with $\lambda < w$ are screened by the electrons in the highly conducting emitter and collector regions. Since such variations do not influence the energy of the well and therefore do not contribute to the electrostatic feedback mechanism which leads to bistability, we obtain $r_{\mathrm{cr}} > w$. However, the condition $r_{\mathrm{cr}} > \ell_{\mathrm{D}}$ is much stronger because $\ell_{\mathrm{D}} \gg w$.

Our estimate $r_{\mathrm{cr}} > \ell_{\mathrm{D}} \sim W \sim 10\ \mu$m suggests that for realistic DBRT parameters the nucleus represents a macroscopic object whose lateral dimension is comparable to the typical lateral size of a DBRT structure.[3] Physically, this results from the efficient re-distribution of electron charge in the quantum well plane. Since the transition probability decreases exponentially with the area of the nucleus,[1,4] the probability of spontaneous appearance of the critical nucleus due to shot noise in the structure with extra-large area $S \gtrsim \pi r_{\mathrm{cr}}^2$ is negligible. We note that the probability of the stochastic generation of a critical nucleus is equal to the probability that a DBRT with a lateral size of $2r_{\mathrm{cr}} \approx 20\ \mu$m is uniformly switched by electron shot noise.

In Ref. 1 a characteristic scale $r_0 = \sqrt{\eta}(\alpha\gamma)^{-1/4}$ was introduced, where r_0 is a characteristic width of the critical profile $n(r)$ (this profile is shown in Fig. 3 in Ref. 1), which corresponds to the saddle point of the functional $F(n)$ (Eq.(14) in Ref. 1). This width is determined by the coefficient of lateral diffusion in the well D ($\eta \sim D$) and the parameters of the effective potential α and γ which reflects the balance of the emitter-well and well-emitter currents (Eq. (5) in Ref. 1). The physical meaning of r_0 is similar to the meaning of r_{cr} in our consideration. It is shown that $r_0 \sim (V_{\mathrm{th}} - V)^{-1}$ and thus $\pi r_0^2 \gg S$ for $(V_{\mathrm{th}} - V) \to 0$.[1] In this case only uniform transitions are possible. The characteristic time of such uniform transitions is given by Eq. (6) in Ref. 1. Since r_0 decreases with increase of $(V_{\mathrm{th}} - V)$, it is assumed that $\pi r_0^2 < S$ for sufficiently large $(V_{\mathrm{th}} - V)$ and then the switching via nucleation becomes possible.[1] Ref. 1 does not provide a lower bound for r_0 in this regime, implicitly assuming that r_0 becomes sufficiently small.

In Ref. 2, which represents an extended and elaborated version of Ref. 1, r_0 is replaced by another characteristic length d. It is related to r_0 as[2]

$$\left(\frac{d}{r_0}\right)^4 = \frac{8\zeta\eta\alpha}{\gamma},$$

where $\zeta \approx 7.75$ (Eq.(57) in Ref. 2). The length d is evaluated as[2]

$$d \sim \frac{1}{\sqrt{n}}\left(\frac{\hbar\sigma}{e^2 T_R}\right)^{3/4}\left(\frac{T_L}{T_R}\right)^{1/2}, \tag{3}$$

where σ is the conductivity in the quantum well and T_L, T_R are transition coefficients of the barriers. For $\sigma = e\,\mu\,n, T_L = T_R \sim \Gamma_R/E_e^F$ and $n \sim \rho_0\,E_e^F$ from (3) we obtain

$$d \sim \left(\frac{\mu\hbar E_e^F}{e\Gamma_R}\right)^{3/4}(\rho_0 E_e^F)^{1/4} = \left(\frac{m\hbar}{\pi e^3}\right)^{1/4}\left(\frac{\mu}{\Gamma_R}\right)^{3/4} E_e^F, \tag{4}$$

where $\rho_0 \equiv m/\pi\hbar^2$ is the two-dimensional density of states. For $\mu \sim 10^5\,\mathrm{cm^2/V\,s}$, $\Gamma_L \sim 1\,\mathrm{meV}$, $E_e^F \sim 10\,\mathrm{meV}$ we obtain $d \sim 1\,\mu$m, in agreement with our estimate based on Eq. (2).

Ref. 2 suggests that the lower bound for d given by Eq.(3) is

$$d \sim \frac{1}{\sqrt{n}} \sim \frac{1}{\sqrt{\rho_o E_e^F}} \approx 20\ \mathrm{nm}. \tag{5}$$

This value corresponds to $T_L = T_R \approx 1$ and the conductivity in the well $\sigma = e^2/\hbar$. Both assumptions are far beyond the limits of applicability of the model under consideration. Indeed, the analysis in Refs. 1,2 is done for the incoherent tunneling regime whereas the limit of transparent barriers $T_L = T_R \approx 1$ clearly corresponds to the coherent tunneling regime. The conductivity $\sigma = e^2/\hbar \sim 2.4\cdot 10^{-4}\ \Omega^{-1}$ corresponds to the low mobility limit when the mean free path of an electron is of the order of the average distance between carriers (effective mobility $\widetilde{\mu} = \sigma/(en) \sim 5\cdot 10^3\ \mathrm{cm^2/Vs}$). Strong electron scattering generally leads to wide broadening of the level in the quantum well, which is known to smooth out the bistability of the resonant-tunneling structure.[13] The broadening Γ_{well} of the quasibound state corresponding to $\sigma = e^2/\hbar$ is of the order of E_e^F which is far too large to observe bistability. Finally, $d = 20$ nm does not meet the condition $d > w$ for the typical width of the

resonant-tunneling structure $w \sim 100$ nm. We conclude that the evaluation of d for $T_{L,R} \to 1$ and $\sigma \to e^2/\hbar$ is physically meaningless.

Our consideration shows that for realistic structural parameters the crossover from uniform stochastic switching to stochastic switching via nucleation might not happen because $r_{\rm cr}^2$ remains comparable to S regardless of the voltage V. In principle, the nucleation scenario remains possible in extra-large structures ($S \gg \pi r_{\rm cr}^2$) when $(V_{\rm th} - V)$ is chosen sufficiently small so that $S \gg \pi r_0^2 > \pi r_{\rm cr}^2$. In practice, in this macroscopic limit the statistical properties of the switching time are determined rather by the fluctuations of the applied voltage V and broadening of the threshold voltage $V_{\rm th}$ due to imperfections of the DBRT structure. The inaccuracy of the applied voltage becomes particularly important for transient measurements like those performed in Ref. 14 when the applied voltage is dynamically increased in a stepwise manner to reach the metastable state at the edge of the bistability range.

In conclusion, our estimates based on the results of the studies of lateral switching fronts in a bistable DBRT[13] suggest that the critical nucleus, which is needed to trigger such a switching front, has a macroscopically large lateral dimension (e.g. ≥ 10 μm) for realistic structural parameters. Therefore it is doubtful that the nucleation scenario which implies triggering of the lateral front by shot electron noise is possible in a DBRT. The effect of electron shot noise on the lifetime of the metastable state rather decreases with increasing area of the DBRT structure and vanishes for structures with macroscopic lateral dimensions, in agreement with Ref. 4. This does not exclude that the nucleation mechanism might be relevant in other bistable systems.

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